PROOFS BY INDUCTION

PHIL 140A SPRING 2016

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1. Prove that $\sum_{k=1}^{n}(2k-1) = n^2$ for all $n \geqslant 1$.

2. Prove that for any set x, if x has exactly n members, then $\mathcal{P}(x)$ has exactly 2^n members (i.e., there are 2^n subsets of x).

3. Define the ***length*** of a propositional formula φ as follows:

$$\begin{aligned} \mathsf{len}(p_i) &= \mathsf{len}(\bot) = 1 \\ \mathsf{len}((\neg\varphi)) &= \mathsf{len}(\varphi) + 1 \\ \mathsf{len}((\varphi \,\Box\, \psi)) &= \mathsf{len}(\varphi) + \mathsf{len}(\psi) + 1. \end{aligned}$$

   Prove that the number of subformulas in φ is less than or equal to $\mathsf{len}(\varphi)$.

4. For any three formulas φ, ψ, and θ, define the formula $\theta[\psi/\varphi]$ to be the result of replacing every instance of ψ in θ with φ (if there is no occurrence of φ in θ, then $\theta[\psi/\varphi] = \theta$). Show that if φ and ψ are logically equivalent (i.e., φ and ψ have the same truth value on every row of the truth table), then θ and $\theta[\psi/\varphi]$ are logically equivalent. (*Hint*: don't forget the case where $\theta = \varphi$.)